Gotta catch 'em all!™

Derivative Rules

1. The Constant Multiple Rule

$$\frac{d}{dx}\Big[k \cdot f(x)\Big] = k \cdot \frac{d}{dx}\Big[f(x)\Big]$$

2. The Sum Rule

$$\frac{d}{dx}\Big[f(x) + g(x)\Big] = \frac{d}{dx}\Big[f(x)\Big] + \frac{d}{dx}\Big[g(x)\Big]$$

3. The Product Rule

$$\frac{d}{dx}\Big[f(x)g(x)\Big] = \frac{d}{dx}\Big[f(x)\Big] \cdot g(x) + f(x) \cdot \frac{d}{dx}\Big[g(x)\Big]$$

4. The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}\Big[f(x)\Big] \cdot g(x) - f(x) \cdot \frac{d}{dx}\Big[g(x)\Big]}{\Big[g(x)\Big]^2}$$

5. The Chain Rule

$$\frac{d}{dx}\Big[f\big(g(x)\big)\Big] = f'\big(g(x)\big) \cdot \frac{d}{dx}\Big[g(x)\Big]$$

The Derivatives of Basic Functions

1. Constants

$$\frac{d}{dx}\Big[k\Big] = 0$$

2. Powers of x

$$\frac{d}{dx}\Big[x^k\Big] = kx^{k-1}$$

3. Sine

$$\frac{d}{dx}\Big[\sin(x)\Big] = \cos(x)$$

4. Cosine

$$\frac{d}{dx}\Big[\cos(x)\Big] = -\sin(x)$$

5. Exponentials

$$\frac{d}{dx}\Big[b^x\Big] = \ln(b) \cdot b^x \qquad \frac{d}{dx}\Big[e^x\Big] = e^x$$

6. Logs

$$\frac{d}{dx}\Big[\log_b(x)\Big] = \frac{1}{\ln(b)} \cdot \frac{1}{x} \qquad \frac{d}{dx}\Big[\ln(x)\Big] = \frac{1}{x}$$

Examples

1. The derivative of any polynomial.

$$\begin{aligned}
&\frac{\mathbf{d}}{\mathbf{dx}}\Big[\mathbf{2x^4+3x^3+x^2-10x-5}\Big] \\
\text{(the Sum Rule)} &= \frac{d}{dx}\Big[2x^4\Big]+\frac{d}{dx}\Big[3x^3\Big]+\frac{d}{dx}\Big[-x^2\Big]+\frac{d}{dx}\Big[-10x\Big]+\frac{d}{dx}\Big[-5\Big] \\
\text{(the Constant Multiple Rule)} &= 2\cdot\frac{d}{dx}\Big[x^4\Big]+3\cdot\frac{d}{dx}\Big[x^3\Big]-\frac{d}{dx}\Big[x^2\Big]-10\cdot\frac{d}{dx}\Big[x\Big]+\frac{d}{dx}\Big[-5\Big] \\
\text{(Derivative of } x^k \text{ and constants)} &= 2\cdot 4x^3+3\cdot 3x^2-2x-10\cdot 1+0 \\
&= \mathbf{8x^3+9x^2-2x-10}
\end{aligned}$$

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2. The derivative of $\tan(x), \sec(x), \csc(x), \cot(x)$
(which can all be written as quotients involving $\sin(x)$ and/or $\cos(x)$).

$$\begin{aligned}
\frac{\mathbf{d}}{\mathbf{dx}}\Big[\mathbf{sec(x)}\Big] &= \frac{d}{dx}\Big[\frac{1}{\cos(x)}\Big] \\
\text{(the Quotient Rule)} &= \frac{\frac{d}{dx}\Big[1\Big]\cdot\cos(x)-1\cdot\frac{d}{dx}\Big[\cos(x)\Big]}{\big[\cos(x)\big]^2} \\
\text{(the Derivative of constants and } \cos(x)\text{)} &= \frac{0\cdot\cos(x)-1\cdot\big[-\sin(x)\big]}{\big[\cos(x)\big]^2} \\
&= \frac{\sin(x)}{\cos^2(x)} = \frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\cos(x)} = \mathbf{sec(x)tan(x)}
\end{aligned}$$

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3. The derivative of any rational function.

$$\begin{aligned}
&\frac{\mathbf{d}}{\mathbf{dx}}\Big[\frac{\mathbf{x^4+2x+1}}{\mathbf{3x^2-2}}\Big] \\
\text{(the Quotient Rule)} &= \frac{\frac{d}{dx}\Big[x^4+2x+1\Big]\cdot(3x^2-2)-(x^4+2x+1)\cdot\frac{d}{dx}\Big[3x^2-2\Big]}{\big[3x^2-2\big]^2} \\
\text{(the Derivative of polynomials (Ex.1))} &= \frac{(4x^3+2)\cdot(3x^2-2)-(x^4+2x+1)\cdot(6x)}{\big[3x^2-2\big]^2} \\
&= (algebra) = \frac{\mathbf{6x^5-8x^3-6x^2-6x-4}}{\mathbf{9x^4-12x+4}}
\end{aligned}$$

4.

$$\mathbf{\frac{d}{dx}\left[(2x+1)^5 \sin(4x)\right]}$$

$$\begin{aligned}
\text{(the Product Rule)} &= \frac{d}{dx}\left[(2x+1)^5\right] \cdot \sin(4x) + (2x+1)^5 \cdot \frac{d}{dx}\left[\sin(4x)\right] \\
\text{(the Chain Rule)} &= \left[5(2x+1)^4 \cdot \frac{d}{dx}[2x+1]\right] \cdot \sin(4x) + (2x+1)^5 \cdot \left[\cos(4x) \cdot \frac{d}{dx}[4x]\right] \\
\text{(basic derivatives)} &= \left[5(2x+1)^4 \cdot 2\right] \cdot \sin(4x) + (2x+1)^5 \cdot \left[\cos(4x) \cdot 4\right] \\
\text{(simplify)} &= 10(2x+1)^4 \sin(4x) + 4(2x+1)^5 \cos(4x)
\end{aligned}$$

5.

$$\mathbf{\frac{d}{dx}\left[\left(x^3 \sin(x) + 2x - 5\right)^{\frac{5}{2}}\right]}$$

$$\begin{aligned}
\text{(the Chain Rule)} &= \frac{5}{2}\left(x^3 \sin(x) + 2x - 5\right)^{\frac{3}{2}} \cdot \frac{d}{dx}\left[x^3 \sin(x) + 2x - 5\right] \\
\text{(the Sum Rule)} &= \frac{5}{2}\left(x^3 \sin(x) + 2x - 5\right)^{\frac{3}{2}} \cdot \left[\frac{d}{dx}\left[x^3 \sin(x)\right] + \frac{d}{dx}[2x-5]\right] \\
\text{(the Product Rule)} &= \frac{5}{2}\left(x^3 \sin(x) + 2x - 5\right)^{\frac{3}{2}} \cdot \left[\frac{d}{dx}\left[x^3\right] \cdot \sin(x) + x^3 \cdot \frac{d}{dx}\left[\sin(x)\right] + \frac{d}{dx}[2x-5]\right] \\
\text{(basic derivatives)} &= \frac{5}{2}\left(x^3 \sin(x) + 2x - 5\right)^{\frac{3}{2}} \cdot \left[3x^2 \cdot \sin(x) + x^3 \cdot \cos(x) + 2\right]
\end{aligned}$$

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6. Basic Chain Rule problems

$$\boxed{\frac{d}{dx}\bigg[f(kx)\bigg] \;=\; f'(kx)\cdot\frac{d}{dx}\bigg[kx\bigg] \;=\; kf'(kx)}$$

$$\frac{d}{dx}\bigg[\sin(5x)\bigg] \;=\; \cos(5x)\cdot\frac{d}{dx}\bigg[5x\bigg] \;=\; \cos(5x)\cdot 5 \;=\; 5\cos(5x)$$

$$\frac{d}{dx}\bigg[e^{2x}\bigg] \;=\; e^{2x}\cdot\frac{d}{dx}\bigg[2x\bigg] \;=\; e^{2x}\cdot 2 \;=\; 2e^{2x}$$

$$\frac{d}{dx}\bigg[\log_3(2x)\bigg] \;=\; \frac{1}{\ln(3)}\cdot\frac{1}{2x}\cdot\frac{d}{dx}\bigg[2x\bigg] \;=\; \frac{1}{\ln(3)}\cdot\frac{1}{2x}\cdot 2 \;=\; \frac{1}{\ln(3)}\cdot\frac{1}{x}$$

$$\boxed{\frac{d}{dx}\bigg[\big(g(x)\big)^n\bigg] \;=\; n\big(g(x)\big)^{n-1}\cdot\frac{d}{dx}\bigg[g(x)\bigg]}$$

$$\frac{d}{dx}\bigg[(x^2+7x+2)^5\bigg] \;=\; 5(x^2+7x+2)^4\cdot\frac{d}{dx}\bigg[x^2+7x+2\bigg] \;=\; 5(x^2+7x+2)^4\cdot(2x+7)$$

$$\frac{d}{dx}\bigg[\sin^5(x)\bigg] \;=\; 5\sin^4(x)\cdot\frac{d}{dx}\bigg[\sin(x)\bigg] \;=\; 5\sin^4(x)\cdot\cos(x)$$

$$\frac{d}{dx}\bigg[(e^x+\ln(x))^5\bigg] \;=\; 5(e^x+\ln(x))^4\cdot\frac{d}{dx}\bigg[e^x+\ln(x)\bigg] \;=\; 5(e^x+\ln(x))^4\cdot\left(e^x+\frac{1}{x}\right)$$