

2 Interlude 3: Slope Fields

3. Choose the differential equation from the list below which corresponds to each of the following slope fields. Write your answer on the line below the slope field.

(i) $\frac{dy}{dx} = x^2 - 2x$

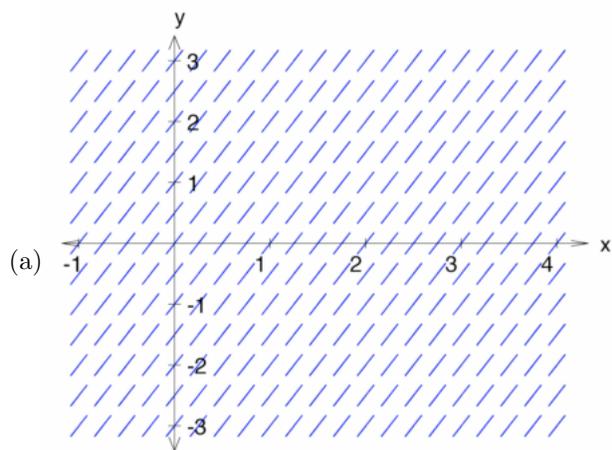
(ii) $\frac{dy}{dx} = y^2 - 2y$

(iii) $\frac{dy}{dx} = 2$

(iv) $\frac{dy}{dx} = 2x - 2$

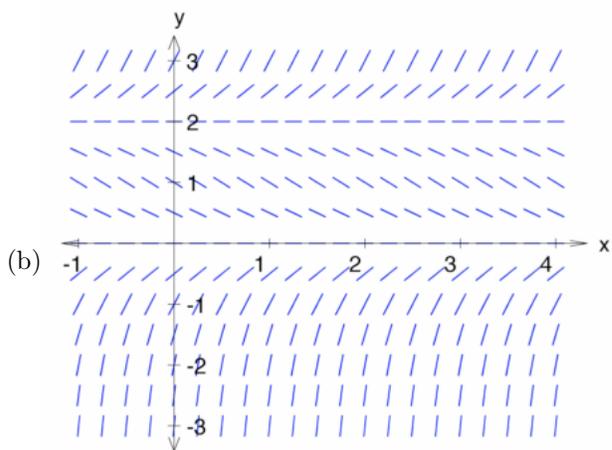
(v) $\frac{dy}{dx} = 2y - 2$

(vi) $\frac{dy}{dx} = 2y$



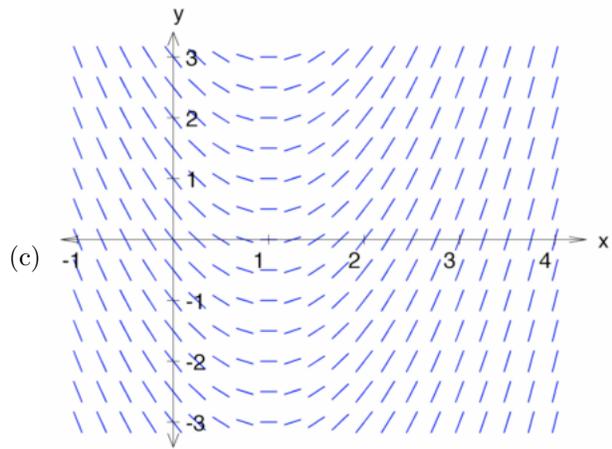
(a)

(a) _____



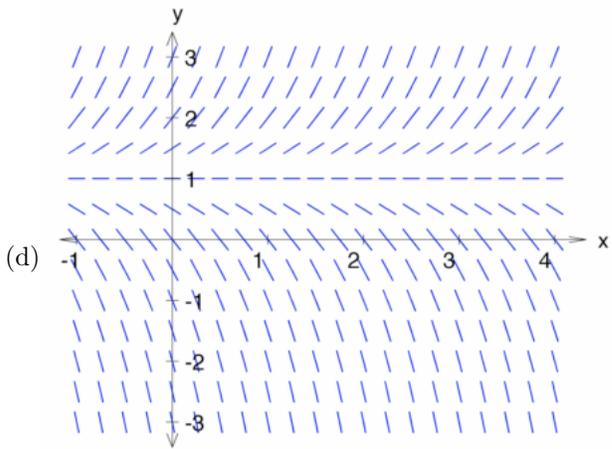
(b)

(b) _____



(c)

(c) _____



(d)

(d) _____

4. For each slope field, sketch the four solutions which correspond to the four initial conditions:

$$y(0) = -1, \quad y(0) = 0, \quad y(0) = 1, \quad y(0) = 2$$

3 Interlude 4: Newton's Law of Cooling

Recall that Newton's Law of Cooling governs the temperature $y(t)$ as a function of time t of an object that is warming or cooling due to its relation to the ambient temperature A , and takes the form of the following differential equation with constant $k > 0$.

$$\frac{dy}{dt} = -k(y - A)$$

5. Which of the following differential equations are instances of Newton's Law of Cooling? For those that are, determine the ambient temperature A .

- (a) $\frac{dy}{dt} = -y$
- (b) $\frac{dy}{dt} = 1 - y$
- (c) $\frac{dy}{dt} = -2(10 - y)$
- (d) $\frac{dy}{dt} = 2(10 - y)$

6. You take an ice-cream out of the freezer, kept at $-4^\circ C$. Outside it is $32^\circ C$. After one minute, the ice-cream has warmed to $1^\circ C$. What is the temperature of the ice-cream after five minutes?

7. A delicious pumpkin pie is removed from an oven that is set at $162^\circ C$ and is placed on a rack to cool. The temperature in the kitchen is $21^\circ C$. Letting $T(t)$ represent the temperature of the pie at any time t (in minutes), Newton's Law of Cooling says that the rate at which the pie cools is described as

$$\frac{dT}{dt} = -k(T - 21), \quad \text{where } T(0) = 162.$$

- (a) Show that the function $T(t) = 21 + 141e^{-kt}$ satisfies the differential equation and the initial value.

- (b) Suppose the temperature of that tasty pie is $91^\circ C$ after 30 minutes of resting on the cool rack. Use part (a) and the new information provided to find the specific formula for the temperature of the pie at time t .