

Math 131 - Study Session Problems

1 3.1 Rolle's Theorem and Mean Value Theorem

1. For each function and interval given below, decide whether Rolle's Theorem applies. If it does, state the conclusion that can be drawn. If it doesn't explain why it doesn't.

(a) $f(x) = x^2 - x$ on the interval $[0, 1]$

(b) $f(x) = x^2 - x$ on the interval $[-1, 1]$

(c) $f(x) = \frac{x^2-4}{x^2}$ on the interval $[-2, 2]$

(d) $f(x) = \frac{x^2-4}{x^2}$ on the interval $[1, 3]$

2. For the same functions and intervals given above, decide whether the Mean Value Theorem applies. If it does, state the conclusion that can be drawn. If it doesn't explain why it doesn't.

2 3.5 Related Rates

1. Suppose the radius r , height h , and volume V of a cylinder are functions of time t . Recall the volume formula for a cylinder is $V = \pi r^2 h$.

(a) How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ if the height of the cylinder remains constant?

(b) How is $\frac{dV}{dt}$ related to $\frac{dh}{dt}$ if the radius of the cylinder remains constant?

(c) How is $\frac{dr}{dt}$ related to $\frac{dh}{dt}$ if the volume of the cylinder remains constant?

- (d) If the height of the cylinder is constantly 10 inches, and the radius is increasing at 2 inches per second, at what rate is the volume changing when the radius is 5 inches?

3 3.2 First Derivative and Curve Sketching

1. For each of the following functions, find the critical points and construct a sign chart for the derivative $f'(x)$. Using the sign chart, determine the intervals on which f is increasing and decreasing.

(a) $f(x) = x^3 - 6x^2 + 9x + 9$

(b) $f(x) = xe^x$

(c) $f(x) = \frac{x}{x^2 + 1}$