Math 131 - Study Session Problems

1. (a) Write the definition of the derivative of $g(x)$ at the point $x = c$.

   (b) Write the equation of the tangent line to the function $g(x)$ at the point $x = c$.

   (c) Use two iterations of Newton's method to approximate the value of x which makes $x^3 + 2x = 1$, starting with an initial guess of $x = 1$.

   (d) How does your answer in (b) relate to Newton's method?

2. For the following compositions of functions $f(g(x))$, identify the outer function $f(x)$, the inner function $g(x)$, and the derivative $\frac{d}{dx}\Big[f(g(x))\Big]$

   (a) $(x^2 + 2x + 1)^5$

   (b) $\sqrt{x^4 + x^2}$

   (c) $(x^4 - 10)^{-3/2}$

   (d) $\sqrt{x + \sqrt{x}}$

3. Find the derivatives of the following functions using derivative rules.

   (a) $\frac{d}{dx}\left[\ln(x^2)\sqrt{x^2+1}\right]$

   (b) $\frac{d}{dx}\left[\dfrac{x^2 + 2^x + 1}{2x^2 + 5x}\right]$

   (c) $\frac{d}{dx}\left[\sqrt{\frac{x+1}{x-1}}\right]$

4. Find $f'(x)$ for each of the following piecewise functions. For each function, sketch the graph of the function and the graph of its derivative. Clearly mark any point where $f(x)$ is not continuous or not differentiable.

   (a) $f(x) = \begin{cases} x^2 - x, & x < 2 \\ 4x - 6, & x \geq 2 \end{cases}$

   (b) $f(x) = \begin{cases} 1 - x, & x \leq 1 \\ 1 + x - x^2, & 1 < x < 2 \\ -1, & x \geq 2 \end{cases}$

5. The equation $x^2 + y^2 + xy = 3$ implicitly defines y as a function of x.

   (a) Verify that the point $(-1, 2)$ is on the graph of this implicit function.

   (b) When $x = 1$, solve for y.

   (c) Use your answer from (b) to explain why this implicit function is not a function of x.

   (d) Find the equation of the tangent line at the point $(-1, 2)$.

   (e) Find all the points where the tangent line is horizontal.

   (f) Find all the points where the tangent line is vertical.