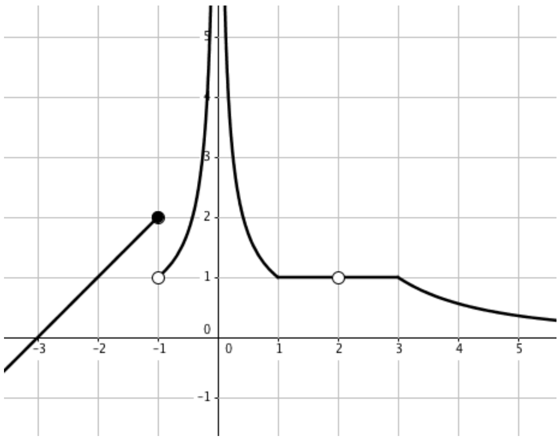
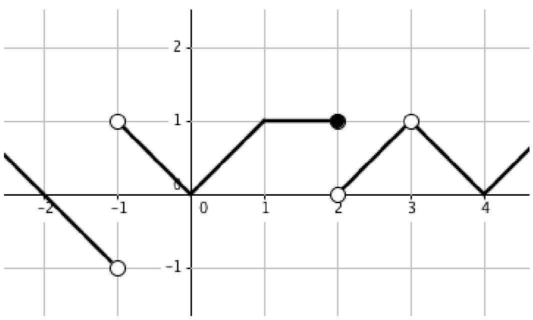
Math 131 - Study Session Problems

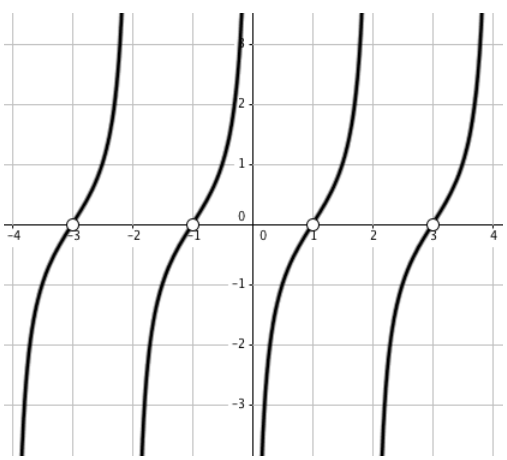
1 Discontinuities

1. For each of the following functions, use the graph to determine where the function fails to be continuous and classify the discontinuity as a removable discontinuity, jump discontinuity, or infinite discontinuity.

   At each discontinuity, determine whether the function is left or right continuous.

   (a) 

   (b) 

   (c) 

   (d) Next to each graph, use limits and the definition of continuity to carefully explain what is happening at $x = 1$ and $x = 2$.

2 Limit Rules

2. Use algebra, limit rules and theorems from class to evaluate the following limits. **Do not look at the graph or make a table.**

   (a) $\displaystyle\lim_{x\to 2} \frac{\frac{4}{x} - 2}{x - 2}$

   (b) $\displaystyle\lim_{x\to 3} \frac{2x - 6}{3 - \sqrt{3x}}$

3 Limits at Infinity

3. Consider the following rational function $\dfrac{x^3 - x + 1}{x^4 + x^2 + 1}$.

   Notice that we can rewrite the function by dividing the numerator and denominator by x^4 (the highest power of x in the denominator).

$$\frac{x^3 - x + 1}{x^4 + x^2 + 1} = \frac{x^3 - x + 1}{x^4 + x^2 + 1}\left(\frac{\frac{1}{x^4}}{\frac{1}{x^4}}\right) = \frac{\frac{1}{x} - \frac{1}{x^3} + \frac{1}{x^4}}{1 + \frac{1}{x^2} + \frac{1}{x^4}}$$

   Use this technique and limit rules to evaluate the following limits.

   (a) $\displaystyle\lim_{x\to\infty} \frac{x^3 - x + 1}{x^4 + x^2 + 1}$

   (b) $\displaystyle\lim_{x\to\infty} \frac{2x^7 + 1}{x^7 + 100x^6}$

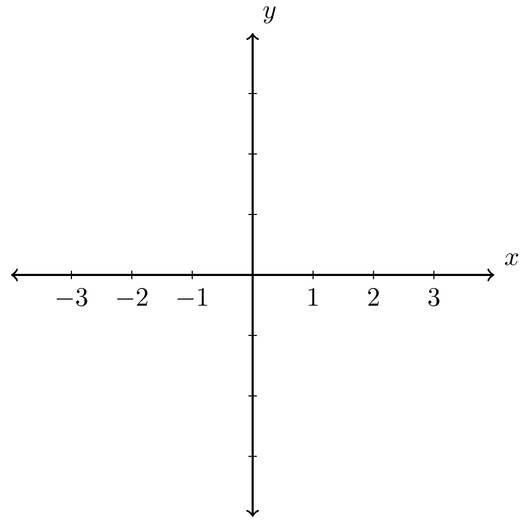
   (c) $\displaystyle\lim_{x\to\infty} \frac{x^3 - 2x^2}{1000x^2 + 1000x}$

4 Sketching Graphs

4. Use the axes below to sketch the graph of a function f with the following properties:

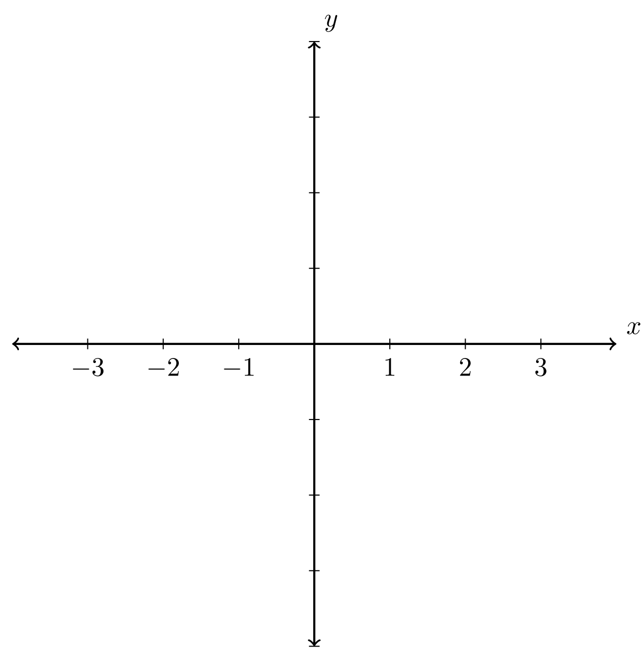
(a)

- f is right continuous everywhere.
- $f(-1) = 2$ and $f(1) = -2$
- f has a vertical asymptote at $x = 2$
- f has no roots



(b)

- $\lim_{x \to -\infty} f(x) = -3$
- f is continuous on the interval $(-\infty, 1)$.
- $f(1) = 0$
- $\lim_{x \to 1+} f(x) = -\infty$
- $\lim_{x \to 2+} f(x) = -1$
- f has a jump discontinuity at $x = 2$.
- f is left-continuous at $x = 2$.



5 Finding Roots

5. Let $f(x) = x^3 - 4x + 2$.

(a) Explain why $f(x)$ is guaranteed to have a root on the interval $[0, 1]$.

(b) Decide on which of the subintervals, $[0, .5], [.5, 1]$ the desired root must be located and explain your reasoning.