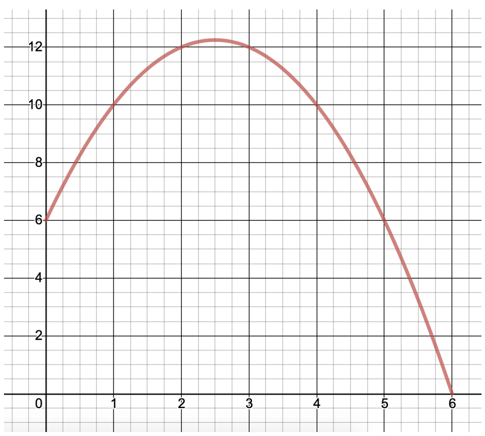
Math 131 - Study Session Problems

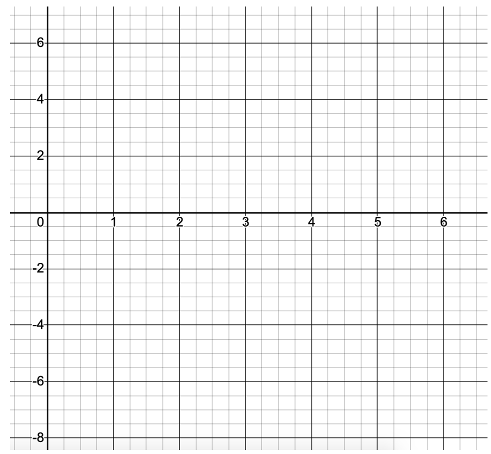
1 Average Rates of Change

Consider the function $s(t) = 6 + 5t - t^2$ where t represents time in seconds, and $y = s(t)$ represents the height in feet above the ground of an apple thrown vertically upwards at time $t = 0$. The domain of s is restricted to the interval $[0, 6]$.

Recall that the **average rate of change (AROC)** of a function f on an interval $[a, b]$ is defined to be $\frac{f(b)-f(a)}{b-a}$. In the case that f is position as a function of time, AROC is called the "average velocity" (since velocity is the rate of change of position with respect to time). In any case, the AROC can be interpreted as the slope of the line joining the points $(a, f(a))$ and $(b, f(b))$.

1. Calculate the AROC of s on the interval $[0, 2]$ and the interval $[5, 6]$. Interpret your results using the graph of s pictured below, and as an average velocity of the apple.





2. Fill in the tables below with the average rates of change of s on the interval $[0, b]$ for a sequence of $b \to 0^+$ on the left, and the average rates of change of s on the interval $[a, 6]$ for a sequence of $a \to 6^-$

b	$s(b)$	AROC of s on $[0, b]$
$b = 1$		
$b = 0.1$		
$b = 0.01$		
$b = 0.001$		

a	$s(a)$	AROC of s on $[a, 6]$
$a = 5$		
$a = 5.9$		
$a = 5.99$		
$a = 5.999$		

3. Use the tables to make an educated guess about the velocity of the apple at time $t = 0$ seconds, and also at time $t = 6$ seconds. What are the units of these numbers? What do their signs mean?

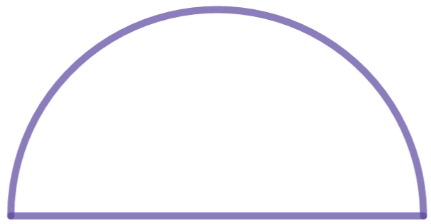
4. Given that $s(t) = 6 + 5t - t^2$, use algebra to find a simplified formula for the function $V(h) = \frac{s(h)-s(0)}{h}$.

5. What is the domain of V? Sketch its graph on the right above.

6. Explain how $\lim_{h \to 0^+} V(h)$ can be determined using the graph of V, and also by using one of the tables above.

2 Delta Epsilon

Consider using a wire of length x cm to manufacture the following semicircular component. Recall that a circle of radius r has circumference $2\pi r$ and area πr^2.



1. If r is the radius of the semicircular component (measured in cm), find a formula for the function $x = P(r)$, that expresses the perimeter x as a function of the radius r.
2. Find a formula for $y = A(r)$, that expresses how the area of the semicircle y in cm^2 is a function of the radius r.
3. Consider the function $f = A \circ P^{-1}$ and explain what the function $y = f(x)$ represents in terms of the variables x and y.
4. If the desired area for the component is $y = 4\ \text{cm}^2$, find the exact length x for the optimal input length.
5. If the acceptable error in the area is $\varepsilon = 0.01\ \text{cm}^2$, so that any component with area outside $(4-\varepsilon, 4+\varepsilon)$ will be rejected, find the largest possible deviation δ from the optimal length that can be allowed.

3 Using tables of function values

1. The following table gives the output of two functions, r and s. For example, the first column of numbers means that $r(-1) = 2$ and $s(-1) = 0$.

x	-1	0	$1/2$	2	3
r	2	-1	1	3	$1/2$
s	0	$1/2$	3.1	-2	4

Using the above table, evaluate the following compositions. If you cannot, explain why.

$r(s(0)) =$ $\qquad$ $s(r(0)) =$ $\qquad$ $s(r(1/2)) =$ $\qquad$ $s(r(r(2))) =$

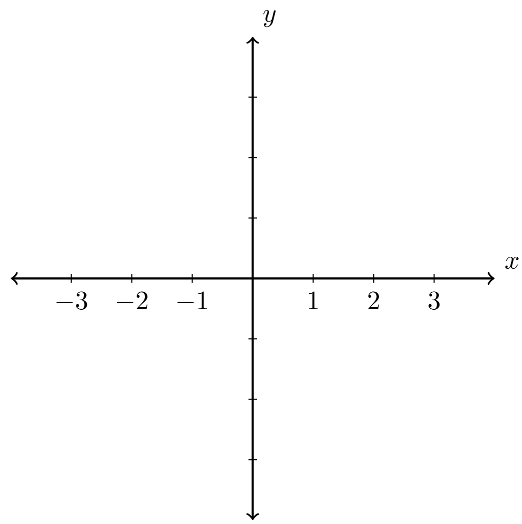
2. Use the completed part of the following table to fill in any blanks.
Hint: fill in the first 3 rows first, and then it should be easy to fill in the remaining rows.

x	-1	0	1	2	3	4
$f(x)$			0	1	0	4
$g(x)$	3	2	0	-1	0	1
$h(x)$	4	3	4		1	-1
$(f \circ f)(x)$	2		2			
$(g \circ h)(x)$				-1		

4 Sketching graphs using limit statements

For each problem, sketch a possible graph of a function with the following properties.

1.
- f is an even function.
- f has a root at $x = 1$.
- f is strictly increasing for $x \in (0, \infty)$
- f has a vertical asymptote at $x = 0$.
- f has a horizontal asymptote at $y = 1$.

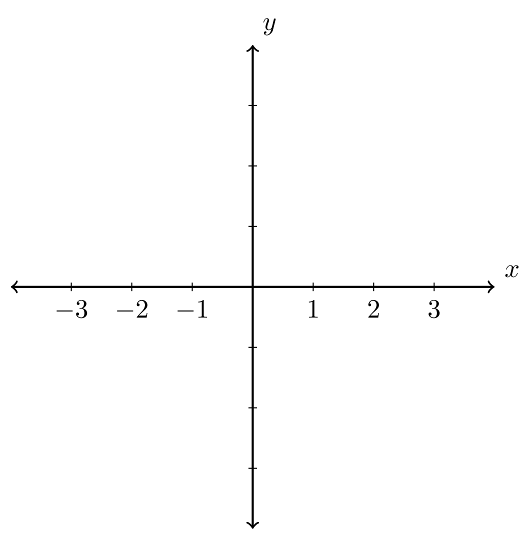


(a) Explain why f has exactly 2 roots.

(b) Determine the value $\lim_{x \to 0} f(x)$. Which of the above properties do you need to justify your answer.

(c) Find a possible formula for $f(x)$ using graph transformations of the function $y = 1/x^2$.

2.
- $\lim_{x \to -1^-} g(x) = 1$
- $\lim_{x \to -1^+} g(x) = 2$
- $\lim_{x \to 2^-} g(x) = -1$
- $\lim_{x \to 2^+} g(x) = -2$
- g has no roots and domain $(-\infty, \infty)$



(a) According to your graph, what are the values of $g(-1)$ and $g(2)$. Did you have to choose them that way?

(b) Determine the value $\lim_{x \to 2} g(x)$. Which of the above properties do you need to justify your answer.